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UNSTEADY FREE CONVECTION FLOW PAST AN INFINITE VERTICAL PLATE WITH VARIABLE SUCTION AND MASS TRANSFER

V. M. SOUNDALGEKAR
Department of Mathematics, Indian Institute of Technology,
Powai, Bombay—400076, India

and

P. D. WAVRE
Department of Mathematics, T.C. College, Baramati (Poona), India

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Abstract—The effects of variable suction on the unsteady two-dimensional free convective flow of a viscous incompressible fluid past an infinite vertical plate have been discussed on taking into consideration the presence of a foreign mass. Approximate solutions to transient flow, the amplitude and the phase of the skin-friction and the rate of heat transfer have been derived. During the course of discussion, the effects of Gr, Gc, Pr, Sc, ω and A (the suction parameter) have been discussed.

NOMENCLATURE

- A, suction parameter;
- $|B|$, amplitude of the skin-friction;
- Ec, Eckert number;
- Gr, Grashof number;
- $|Q|$, amplitude of the rate of heat transfer;
- t, dimensionless time;
- u, dimensionless velocity;
- v_0, suction velocity;
- y, dimensionless co-ordinate normal to the wall;
- ω, dimensionless frequency;
- C, non-dimensional species concentration;
- Sc, Schmidt number;
- θ, dimensionless temperature;
- Gc, modified Grashof number;
- α, phase angle of skin-friction;
- β, phase angle of rate of heat transfer.

1. INTRODUCTION

In the previous paper by the authors [2], the details of the problem from the physical point of view have been described. We now propose to study in this paper the effects of variable suction on the unsteady free convective flow in the presence of mass transfer. In Section 2, following the earlier paper by the authors [2] and Soundalgekar [1], the equations governing the flow are expressed in non-dimensional form. The solutions being lengthy in character, to save space, they are not mentioned. As the variable suction affects the unsteady part, the study of transient velocity, the transient temperature, the amplitude and the phase of the skin-friction and the rate of heat transfer is presented in Section 2 and in Section 3, the conclusions are set out.

2. MATHEMATICAL ANALYSIS

The geometry and the assumptions implied in this problem have been explained [2]. So following

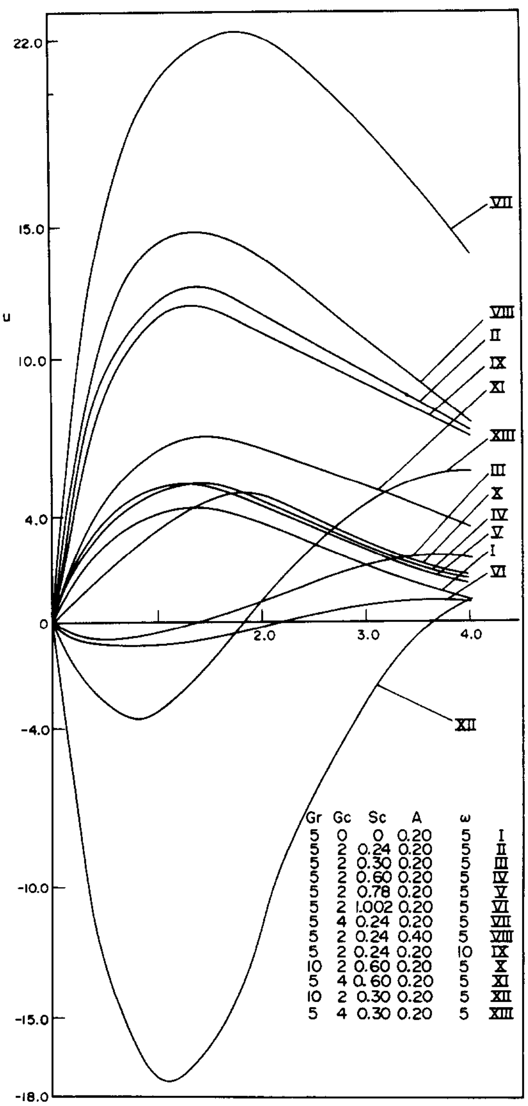

FIG. 1. Transient velocity profiles. $Pr = 0.71$, $Ec = 0.01$, $\omega t = \pi/2$, $\varepsilon = 0.2$.

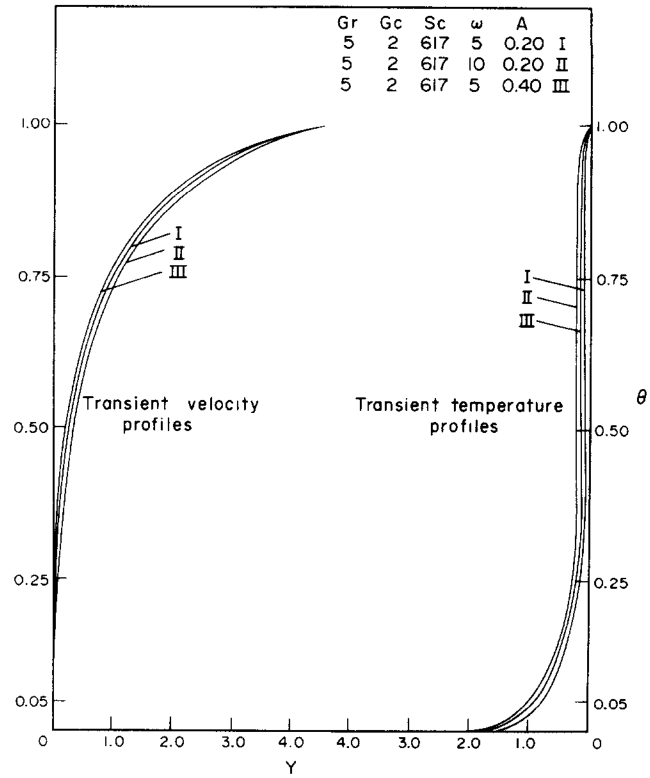

FIG. 2. Transient profiles. $Pr = 7$, $Ec = 0.01$, $\omega t = 0.01$, $\varepsilon = 0.2$.

Soundalgekar [1] it can be shown that the problem is governed by the following set of equations in non-dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon A\,e^{i\omega t})\frac{\partial u}{\partial y} = Gr\theta + GcC + \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\frac{Pr}{4}\frac{\partial \theta}{\partial t} - Pr(1+\varepsilon A\,e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PrEc\left(\frac{\partial u}{\partial y}\right)^2 \quad (2)$$

$$\frac{Sc}{4}\frac{\partial C}{\partial t} - Sc(1+\varepsilon A\,e^{i\omega t})\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} \quad (3)$$

and the boundary conditions are

$$\begin{aligned} &u = 0,\ \theta = \theta_w = (1+\varepsilon e^{i\omega t}),\ C = 1 \quad \text{at } y = 0 \\ &u = 0,\ \theta = 0,\ C = 0 \quad \text{as } y \to \infty. \end{aligned} \quad (4)$$

Following the method of the earlier paper, we have derived all the solutions. The mean flow is not affected by the variable suction. So the expressions for the transient velocity and the transient temperature are derived and they are shown on Figs. 1–3. To save space, the expressions are not mentioned here. It is rather surprising to note the effects of the variable suction on the transient velocity. We compare the curves in Fig. 1 with those of Fig. 4 of the earlier paper. Comparing curves II from these two figures, for H_2, we conclude that there is a rise in the transient velocity in the presence of variable suction. However, curves III from these figures for He are quite interesting. In the presence of variable suction, the transient velocity near the plate is observed to be negative and hence we conclude that there may occur separation near the plate when He is present. The same is true for CO_2, but in the presence of H_2O and NH_3, there is observed to be a rise in the transient velocity. Thus, at small values of A, separated type of flow may exists near the plate in the presence of He and CO_2. In the presence of H_2, an increase in A leads to an increase in the transient velocity. An increase in Gr or Gc in the presence of He is to enhance separation near the plate. But in the presence of H_2O, when $A \neq 0$, an increase in Gr or Gc leads to a rise in the transient velocity. In water (Fig. 2) we observe that an increase in A leads to an increase in the transient velocity.

In Fig. 3, the transient temperature profiles are shown for air. When $A \neq 0$, as compared to the case of $Gc = 0$, in the presence of a foreign mass, the transient temperature is always high, and it is maximum when CO_2 is present. In the presence of H_2, the transient temperature increases with increasing A. An increase in Gr or Gc also leads to an increase in the transient temperature when $A \neq 0$. In water (Fig. 2), it increases with increasing A.

As in [2], we have calculated the amplitude $|B|$ and phase $\tan\alpha$ of the skin-friction. The numerical values of $|B|$ and $\tan\alpha$ are entered in Tables 1 and 2 respectively. We observe from this table that in the presence of a variable suction, $|B|$ for air, increases sharply when a foreign mass is present, and it is very high when CO_2 is present. An increase in Gr or Gc also leads to an increase in the value of $|B|$. This is not the case in water. We observe that due to variable suction $|B|$ decreases, and it decreases more with increasing A. For air there is always a phase-lag when He is present. Also there is phase-lag at high values of ω when CO_2 or

Table 1. Values of the amplitude $|B|$ of the skin friction

				$Ec = 0.01$			
Pr	Gr	Gc	A	Sc/ω	5	10	15
0.71	5	0	0.20	0	25.2	14.19	10.002
0.71	5	2	0.20	0.24	40.7	17.56	12.85
				0.30	187.12	137.67	114.31
				0.60	60.83	48.87	45.95
				0.78	52.43	38.78	32.01
				1.002	9682.1	7835.4	5779.5
0.71	5	4	0.20	0.24	65.09	39.58	33.99
				0.30	365.3	366.55	220.12
				0.60	82.52	60.64	53.51
				0.78	77.21	54.45	42.77
				1.002	40938.0	32824.0	24153.0
0.71	10	2	0.20	0.24	241.23	73.21	38.09
				0.30	774.68	565.72	472.46
				0.60	411.15	345.66	336.83
				0.78	337.28	250.44	211.28
				1.002	17334.0	14731.0	11090.0
0.71	5	2	0.40	0.24	24.713	28.91	22.89
				0.30	381.33	279.84	231.47
				0.60	119.08	96.66	90.55
				0.78	100.45	76.49	63.10
				1.002	19477.0	15666.0	11542.0
7.0	5	2	0.20	100	0.43044	0.41913	0.39489
				617	0.43034	0.41909	0.39486
7.0	5	4	0.20	100	0.43045	0.41918	0.39495
				617	0.43034	0.41909	0.39486
7.0	10	2	0.20	100	0.95926	0.92891	0.87025
				617	0.95930	0.92894	0.87028
7.0	5	2	0.40	100	0.25251	0.29053	0.29565
				617	0.25244	0.29046	0.29558

Table 2. Values of the $\tan\alpha$ phase of the skin friction

				$Ec = 0.01$			
Pr	Gr	Gc	A	Sc/ω	5	10	15
0.71	5	0	0.20	0	11.79	16.08	47.89
0.71	5	2	0.20	0.24	0.735	0.017	−0.681
				0.30	−0.944	−0.832	−0.751
				0.60	2.15	3.26	3.06
				0.78	1.42	3.18	4.80
				1.002	2.008	−5.401	−1.734
0.71	5	4	0.20	0.24	0.469	−0.222	−0.600
				0.30	−0.848	−0.814	−0.789
				0.60	2.687	5.143	4.640
				0.78	1.593	4.547	9.582
				1.002	1.919	−6.423	−1.886
0.71	10	2	0.20	0.24	1.0344	0.3579	−1.235
				0.30	−1.162	−8.616	−6.780
				0.60	1.667	2.289	2.243
				0.78	1.134	2.101	2.744
				1.002	2.528	−3.565	−1.412
0.71	5	2	0.40	0.24	0.869	0.616	−0.883
				0.30	−0.916	−0.807	−0.732
				0.60	2.621	4.173	3.582
				0.78	1.705	4.369	7.181
				1.002	1.967	−0.564	−1.764
7.0	5	2	0.20	100	−0.16404	−0.27793	−0.34661
				617	−0.16471	−0.27794	−0.34663
7.0	5	4	0.20	100	−0.16470	−0.27791	−0.34658
				617	−0.16471	−0.27794	−0.34663
7.0	10	2	0.20	100	−0.23556	−0.33915	−0.39849
				617	−0.23557	−0.33915	−0.39849
7.0	5	2	0.40	100	0.17036	−0.59413	−0.10058
				617	0.17047	−0.00589	−0.10056

Table 3. Values of $|Q|$ the amplitude of the rate of heat transfer

				$Ec = 0.01$			
Pr	Gr	Gc	A	Sc/ω	5	10	15
0.71	5	0	0.20	0	2.70	2.90	2.88
0.71	5	2	0.20	0.24	3.44	3.71	3.46
				0.30	3.35	3.59	3.38
				0.60	3.28	3.50	3.31
				0.78	3.51	3.75	3.49
				1.002	231.20	779.86	359.82
0.71	5	4	0.20	0.24	4.15	4.62	4.09
				0.30	3.97	4.37	3.92
				0.60	3.85	4.17	3.78
				0.78	4.31	4.75	4.17
				1.002	91.74	318.77	145.86
0.71	10	2	0.20	0.24	9.07	8.48	7.49
				0.30	8.89	8.27	7.33
				0.60	8.77	8.09	7.18
				0.78	9.21	8.54	7.53
				1.002	244.79	748.65	353.48
0.71	5	2	0.40	0.24	3.01	4.00	3.39
				0.30	2.94	3.84	3.30
				0.60	2.91	3.71	3.23
				0.78	3.10	4.09	3.43
				1.002	444.55	1552.7	710.22
7.0	5	2	0.20	100	8.4950	8.9115	9.3992
				617	8.4954	8.9110	9.3988
7.0	5	4	0.20	100	8.4959	8.9114	9.3991
				617	8.4953	8.9110	9.3988
7.0	10	2	0.20	100	8.4182	8.8506	9.3517
				617	8.4157	8.8489	9.3505
7.0	5	2	0.40	100	9.8111	10.045	10.367
				617	9.8105	10.044	10.367

Table 4. Values of $\tan\beta$, the phase of rate of heat transfer

				$Ec = 0.01$			
Pr	Gr	Gc	A	Sc/ω	5	10	15
0.71	5	0	0.20	0	−0.091	0.253	0.258
0.71	5	2	0.20	0.24	−0.045	0.285	0.218
				0.30	−0.046	0.279	0.221
				0.60	−0.045	0.275	0.224
				0.78	−0.046	0.289	0.216
				1.002	−3.110	3.672	3.973
0.71	5	4	0.20	0.24	−0.038	0.345	0.209
				0.30	−0.042	0.328	0.209
				0.60	−0.045	0.313	0.209
				0.78	−0.038	0.352	0.207
				1.002	8.308	3.234	3.096
0.71	10	2	0.20	0.24	−0.258	0.083	−0.054
				0.30	−0.262	0.076	−0.055
				0.60	−0.265	0.070	−0.057
				0.78	−0.256	0.086	−0.054
				1.002	−3.786	5.888	0.391
0.71	5	2	0.40	0.24	−0.032	0.590	0.401
				0.30	−0.042	0.568	0.395
				0.60	−0.051	0.550	0.389
				0.78	−0.030	0.598	0.403
				1.002	6.430	3.137	2.924
7.0	5	2	0.20	100	0.11541	0.20824	0.27856
				617	0.11557	0.20823	0.27855
7.0	5	4	0.20	100	0.11558	0.20824	0.27856
				617	0.11557	0.20823	0.27855
7.0	10	2	0.20	100	0.11786	0.21113	0.28172
				617	0.11782	0.21110	0.28170
7.0	5	2	0.40	100	0.0721	0.14898	0.20998
				617	0.07719	0.14896	0.20997

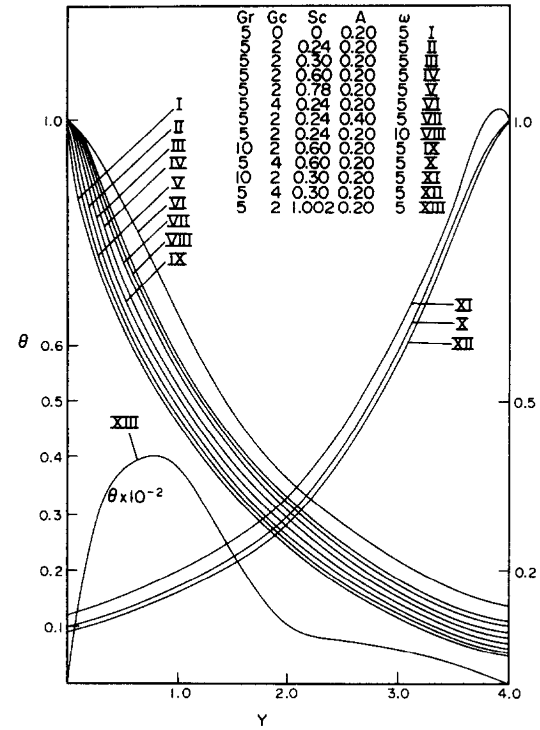

FIG. 3. Transient temperature profiles. $Pr = 0{,}71$, $Ec = 0.01$, $\omega t = \pi/2$, $\varepsilon = 0.2$.

H_2 is present. Otherwise there is always a phase lead. However in case of water even in the presence of variable suction there always exists a phase-lag.

As in [2], we have calculated the amplitude $|Q|$ and the phase $\tan\beta$ of the rate of heat transfer. Their values are entered in Tables 3 and 4 respectively. It has been observed in [2] that in case of air for constant suction the amplitude $|Q|$ decreases with increasing ω. But in the presence of variable suction $|Q|$ increases when ω increases from 5 to 10 and with further increase in ω we observe that $|Q|$ decreases. An increase in A leads to a decrease in the value of $|Q|$ when $Sc < 1$ whereas in the presence of CO_2 the value of $|Q|$ increases very sharply due to an increase in A. An increase in Gr or Gc leads to an increase in $|Q|$. In the case of water $|Q|$ behaves in quite a different way as compared to that of air. We observe in this case that $|Q|$ increases due to variable suction and it increases more due to increasing A. From Table 4 we observe that for small values of Gr and ω and for any value of Gc and A there is a phase-lag, when $Sc < 1$. But for large values of Gc and A, in the presence of CO_2 there is always a phase-lead. Except for large values of Gr there is always a phase-lead when ω is increased. In case of water there is always a phase-lead.

3. CONCLUSIONS

1. In the presence of H_2, H_2O and CO_2 the transient velocity increases in the presence of variable suction but in the presence of He or CO_2 the separated type of flow exists near the plate, even though $A \neq 0$.
2. An increase in Gr or Gc when $A \neq 0$, enhances separation in the presence of He but leads to an increase in the transient velocity in other areas.
3. For water the transient velocity increases with increasing A.
4. For $A \neq 0$ the transient temperature increases due to the presence of a foreign mass and the increase is considerably high when CO_2 is present.
5. An increase in A, Gr, or Gc leads to an increase in the transient temperature both in the case of air and water.
6. $|B|$ for air increases sharply when $A \neq 0$ and the foreign mass is present especially for CO_2 it is very high. It also increases with increasing Gr or Gc.
7. In water, $|B|$ decreases when $A \neq 0$ and decreases more with increasing A.
8. For air, $|Q|$ decreases with increasing A when $Sc < 1$ and increases sharply when $Sc \sim 1$. An increase in Gr or Gc leads to an increase in $|Q|$.
9. In water, $|Q|$ increases when $A \neq 0$ and increases more with increasing A.

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CONVECTION LIBRE INSTATIONNAIRE AUTOUR D'UNE PLAQUE VERTICALE ET INFINIE AVEC UN TRANSFERT MASSIQUE PAR ASPIRATION VARIABLE

Résumé—On considère les effets de l'aspiration variable sur la convection naturelle instationnaire et bidimensionnelle d'un fluide visqueux incompressible autour d'une plaque verticale et infinie, en prenant en considération la présence d'une matière étrangère. On obtient des solutions approchés de l'écoulement variable, de l'amplitude et de la phase du frottement pariétal et du flux thermique. On discute l'influence de Gr, Gc, Pr, Sc, ω et A (paramètre d'aspiration).

INSTATIONÄRE, FREIE KONVEKTION AN EINER UNENDLICH AUSGEDEHNTEN, VERTIKALEN PLATTE MIT VERÄNDERLICHER ABSAUGUNG UND STOFFÜBERGANG

Zusammenfassung—Der Einfluß einer veränderlichen Absaugung auf die zweidimensionale, instationäre freie Konvektionsströmung eines zähen, inkompressiblen Fluids an einer unendlich ausgedehnten, vertikalen Platte wurde unter Berücksichtigung der Anwesenheit von Fremdstoffen untersucht. Es wurden Näherungslösungen für den Verlauf von Strömung und Wandreibung sowie für den Wärmeübergang hergeleitet. Der Einfluß von Gr, Gc, Pr, Ec, Sc, ω und A (Absaugparameter) wird diskutiert.

НЕСТАЦИОНАРНОЕ ОБТЕКАНИЕ БЕСКОНЕЧНОЙ ВЕРТИКАЛЬНОЙ ПЛАСТИНЫ В УСЛОВИЯХ СВОБОДНОЙ КОНВЕКЦИИ ПРИ ПЕРЕМЕННОМ ОТСОСЕ И МАССОПЕРЕНОСЕ

Аннотация — Рассматривается явление переменного отсоса при нестационарном обтекании бесконечной вертикальной пластины двумерным потоком несжимаемой вязкой жидкости в условиях свободной конвекции при наличии инородной массы. Получены приближенные решения для потока в переходном режиме, амплитуды и фазы поверхностного трения, а также для теплового потока. Обсуждается влияние чисел Gr, Gc, Pr, Sc, ω и A (параметра вдува).